reduced by introducing polymer additives. These additives can be chosen so as to decrease at the same time the heat transferred from the mud.

NOTATION

Here, x and t are the spatial coordinate and the time; $x_m(t)$, $x_s(t)$ are the coordinates of the erosion and thaw fronts; $x_a(t)$ is the coordinate of the ablation front in the standard formulation; T_i is the initial temperature of the porous medium; T_s is the melting point of the interstitial ice; T_m is the temperature of the soil framework, corresponding to a fixed degree of saturation with water σ_m; $\sigma(x, t)$ is the degree of saturation of the pore space between x_m and x_s with water; $\sigma_m = \sigma(x_m, t)$, λ_w, λ_i, λ_{fw} are the thermal conductivity of ice, water, and the soil framework; $(c\rho)_i$, $(c\rho)_w$, $(c\rho)_{fw}$ are the corresponding volume heat capacities; ρ_i is the density of ice; L is the specific heat of melting of ice; and, m is the porosity of the soil.

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SOLUTION OF THE HEAT CONDUCTION PROBLEM FOR LAMINAR ORTHOTROPIC SLABS IN A SPATIAL FORMULATION

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A solution is obtained for the stationary heat conduction problem in a spatial formulation for rectangular slabs with an arbitrary quantity of orthotropic layers.

A survey of investigations in the area of the analysis of laminar slabs under thermal actions showed that there are no solutions in a three-dimensional formulation for slabs with anisotropic (orthotropic) layers.

A laminar slab in a stationary temperature field is examined in this paper. The slab is referred to a rectangular x_1,x_2,x_3 coordinate system. The problem is solved by the conjugatuion method [1]. Zero temperature is maintained on the slab side surfaces, i.e., we have T = 0 for $x_1 = 0, a_1$ and $x_2 = 0, a_2$. The following boundary conditions [2] are possible on the slab face surfaces ($x_3 = b^{(\ell)}$, $\ell = 0, n$):

1) First kind

$$T(x_i,\ b^{(l)}) = f^{(l)}(x_i),\ l = 0,\ n;\ i = 1,\ 2; \tag{1}$$

2) Second kind

$$\lambda_3^{(1)} T_{,3}(x_i,\ b^{(0)}) = q^{(0)}(x_i);\ \ \lambda_3^{(n)} T_{,3}(x_i,\ b^{(n)}) = q^{(n)}(x_i); \tag{2}$$

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TABLE 1. Comparison of the Results of a Computation for a Sandwich Slab for Boundary Conditions of the First Kind

$\bar{z}=8x_3/h$	$a/h=5$		$a/h=10$	
	proposed	[3]	proposed	[3]
4	80,00	80,00	80,00	80,00
3	58,23	60,20	66,96	67,11
2	42,51	45,99	55,66	55,92
1	31,21	35,64	45,78	46,16
0	23,15	27,89	37,09	37,51
—1	17,50	21,81	29,36	29,75
—2	13,66	16,89	22,38	22,68
—3	11,25	12,90	15,99	16,14
—4	10,00	10,00	10,00	10,00

TABLE 2. Comparison of the Results of a Computation for a Sandwich Slab for Different Boundary Conditions on the Face Surfaces

$\bar{z}=10x_3/h$	Boundary conditions on the face surfaces					
	(12)		(13)		(14)	
	proposed	[4]	proposed	[4]	proposed	[4]
5	50,00	50,00	66,77	66,76	48,91	48,91
4	48,84	48,90	65,20	65,26	47,77	47,83
2	33,11	32,13	41,10	40,03	32,25	31,30
0	23,22	21,83	24,26	22,81	22,41	21,07
—2	17,42	16,53	11,69	10,80	16,52	15,67
—4	14,69	14,75	1,181	1,236	13,55	13,61
—5	15,00	15,00	1,156	1,146	13,83	13,83

3) Third kind

$$\lambda_3^{(1)}T_{,3}(x_i,\ b^{(0)}) = \alpha^{(0)}(T_c^{(0)}(x_i) + T(x_i,\ b^{(0)}));$$
$$\lambda_3^{(n)}T_{,3}(x_i,\ b^{(n)}) = \alpha^{(n)}(T_c^{(n)}(x_i) + T(x_i,\ b^{(n)})). \tag{3}$$

Differentiation is denoted by the common in the subscripts in (1)-(3) and later, the given functions allow of expansion in double trigonometric Fourier series, and ideal thermal contact conditions are assured between the layers (boundary condition of the fourth kind):

$$T^{(k-1)}(x_i,\ b^{(k-1)}) = T^{(k)}(x_i,\ b^{(k-1)});$$
$$\lambda_3^{(k-1)}T_{,3}^{(k-1)}(x_i,\ b^{(k-1)}) = \lambda_3^{(k)}T_{,3}^{(k)}(x_i,\ b^{(k-1)}). \tag{4}$$

It is assured that the temperature field in the slab satisfied the stationary heat conduction equation (no internal heat sources) [2]

$$\lambda_1 T_{,11} + \lambda_2 T_{,22} + \lambda_3 T_{,33} = 0. \tag{5}$$

The desired temperature distribution over the domain of the k-th layer of the slab thickness $h^{(k)}$ is represented in the form of the double trigonometric Fourier series

$$T(x_i,\ x_3) = \sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty} T^*(x_3)\sin(\alpha x_1)\sin(\beta x_2), \tag{6}$$

where

$$T^*(x_3) = F\exp(\rho x_3). \tag{7}$$

Here $\alpha = m_1\pi/a_1$; $\beta = m_2\pi/a_2$; $\rho = \gamma\pi/h^{(k)}$; F are desired coefficients and γ is a desired quantity.

Representation of the temperature in the form (6) permits satisfying the boundary conditions on the slab side surfaces that are maintained at the zero temperature.

Substitution of (6) into (5) results in an equation of the form

$$(-\lambda_1\alpha^2 - \lambda_2\beta^2 + \lambda_3\rho^2)F = 0. \tag{8}$$

Roots of the characteristic equation

$$\rho = \pm \sqrt{(\lambda_1\alpha^2 + \lambda_2\beta^2)/\lambda_3} \quad (9)$$

follow from (8), and therefore, $\gamma = h^{(k)}\rho/\pi$.

Let us note that the quantity ρ is a real number since all the components in (9) are positive numbers.

Let us represent the expansion of the temperature over the layer thickness in the form

$$T^*(x_3) = F_{11}\operatorname{sh} f + F_{12} f \operatorname{sh} f + F_{13}\operatorname{ch} f + F_{14} f \operatorname{ch} f, \quad (10)$$

where $f = \gamma\pi x_3/h^{(k)}$.

Let us note that the heat conduction equation (5) is valid for any point within the slab. Substitution of (10) into (5) results in $F_{12} = F_{14} = 0$. And, finally, within each layer the temperature over the thickness is described by the law

$$T^*(x_3) = F_{11}\operatorname{sh} f + F_{13}\operatorname{ch} f. \quad (11)$$

The expansion (11) includes two desired coefficients F_{11} and F_{13} for each layer. We have 2n coefficients for a slab of n layers.

To determine them we must satisfy two boundary conditions on the face surfaces: (1), (2), (3) and 2(n − 1) conditions of ideal thermal contact on the layer interfacial surfaces (4). We finally obtain 2 + 2(n − 1) = 2n required equations. The algebraic equations obtained are linearly independent since the expansion functions are nonlinear over the thickness. The solution of the system of 2n linear algebraic equations permits finding the desired expansion coefficients F_{11} and F_{13} for each layer and describing the temperature distribution over the slab thickness at an arbitrary point with the coordinates x_1, x_2.

The algorithm of the solution of the spatial heat conduction problem for the class of problems considered is realized in the form of a program packet for the ES electronic computer.

We make certain comparisons with other solutions to give a foundation for the reliability of the solution constructed.

Eample 1. Let us examine a square sandwich slab ($a_1 = a_2 = a$) on whose face surfaces the temperature distribution

$$T(x_i, b^{(l)}) = T_l \sin(\alpha x_1)\sin(\alpha x_2), \quad (12)$$

is given where $\alpha = \pi/a$; T_0 = 10 degrees and T_n = 80 degrees. The heat conduction coefficients of the layer materials are the following

$$(\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)}) = (14{,}6;\ 0{,}93;\ 0{,}93)\ \text{W/(m·deg)} \quad (k = 1, 3);$$

$$(\lambda_1^{(2)}, \lambda_2^{(2)}, \lambda_3^{(2)}) = (0{,}93;\ 14{,}6;\ 0{,}93)\ \text{W/(m·deg)}.$$

The layer thicknesses are $h^{(k)}$ = (0.25, 0.5, 0.25)h, where h is the total thickness of the slab. The results of a computation obtained on the basis of the proposed approach and using the modified alternate-triangle method [3] are represented in Table 1.

Example 2. A square (a × a) sandwich slab on whose face surfaces are given the temperature distribution (12) and the thermal flux intensity

$$q^{(l)}(x_i) = \bar{q}_l \sin(\alpha x_1)\sin(\alpha x_2), \quad (13)$$

as well as the environment temperature

$$T_c^{(l)}(x_i) = \bar{T}_c^{(l)} \sin(\alpha x_1)\sin(\alpha x_2), \quad (14)$$

where $\alpha = \pi/a$; $\bar{q}_n$ = 30 W/m²; $\bar{q}_0$ = 0; $\bar{T}_c^{(0)}$= 15 degree; $\bar{T}_c^{(n)}$ = 50 degree; ℓ = 0, n. The heat elimination coefficients are $(\alpha^{(0)}, \alpha^{(n)})$ = (5; 20) W/m²·deg). The heat conduction coefficients of the layer materials are $(\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)})$ = (10; 1; 1) W/(m·deg); $\lambda_p^{(2)} = 10^{-2}\lambda_p^{(1)}$(k = 1, 3; p = 1, 2, 3). The thickness of the layers are $h^{(k)}$ = (0.1, 0.8, 0.1)h, where h = 1 m is the total slab thickness. The computation is carried out for a/h = 5. Results of solving these problems, obtained on the basis of the proposed approach and by using a refined model based on application of a nonlinear temperature distribution law [4] are presented in Table 2.

The approach developed to the solution of heat conduction problem in a three-dimensional formulation is "exact" to a definite extent, the error is due just to the numerical realization. Results obtained on its basis can be test data for the foundation of the reliability of different approximate models applied in the computation of laminar orthotropic slabs under thermal action.

NOTATION

λ_j (j = 1, 2, 3) and heat conduction coefficients of an orthotropic body; $T(x_i, b^{(\ell)})$ are temperatures on the slab face surfaces; q is the heat flux; $\alpha^{(\ell)}$ is the heat elimination coefficient; $T_c^{(\ell)}$ is the environment function, and ℓ = 0, n.

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MATHEMATICAL MODELING OF THE PROCESS OF PLANE POLYMER FILM FORMATION

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The approximate solution is considered of a nonlinear nostationary problem of heat conduction with conjugate boundary conditions.

A number of papers [1, 2] that take account most completely of the factors exerting substantial influence on the progress of a process is devoted to the mathematical modeling of polymer film and fiber formation. However, the assumptions made therein do not permit description of the film formation on cooling rollers with the accuracy needed for practical purposes. The problem is solved for boundary conditions of the third kind in all the papers mentioned, including in [1-3] for symmetric boundary conditions. This does not correspond to the actual technological process since cooling from the rollers is considerably more intense than from the air. Moreover, the assumption about boundary conditions of the third kind for the heat transmission through the roller wall does not permit taking account of the nonlinearity of the temperature profile therein during heating, which results in substantial errors in the initial cooling period. In this case conjugate heat transfer conditions assuming the interdependence of the film and roller wall temperature fields must be given on the film-roller boundary. The dependence of the thermophysical properties on the degree of crystallinity is not taken into account in [1] and on the temperature in [2, 3]. Utilization of finite-difference methods [1-3] can cause significant difficulties in the practice of engineering optimization computations.

The problem of heat transfer and nonisothermal crystallization of polymer films on a cooling roller system is solved in this paper by linearization and use of the Foureri, Galerkin–Kantorovich, and Kutta–Merson methods. A number of assumptions is made is the construction of the mathematical model of the process. A one-dimensional problem is considered since the roller diameter and film width exceed its thickness significantly. The initial temperature of the polymer melt is constant and homogeneous, and the initial roller wall temperature are constant and equal to the cooling fluid temperatures. The polymer thermophysical properties are described by using a two-phase model and depend linearly on

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